Coastal Engineering Technical Note

CALCULATING TIDAL ELEVATION PROBABILITIES

PURPOSE: To provide design guidance by pointing out the types of tidal elevation probability estimates that can be performed using CERC Special Report SR-7, and to illustrate the use of the probability tables through several examples.

INTRODUCTION: Calculation of tidal elevation probabilities for the 55 United States tide stations listed in the Table is performed using *Tides and Tidal Datums in the United States* (SR-7) by Harris (1981). The probability tables in SR-7 have been prepared using a computer program which calculates statistical distributions of tidal height variations at each station.

The tide probability tables in Appendix B of SR-7 provide a means of estimating such statistical information as:

1. The extreme highest and extreme lowest tidal water level expected to occur in a 19-year period (Table a).
2. The probability that the tide level will be higher than (or less than) a given level at some time during a calendar month (Table a).
3. The probability that a daily high or low water level will exceed a given level (Table b).
4. The probability that a daily higher high or lower low water level will exceed a given level (Table b).
5. The probability that an hourly tide level will exceed a given level (Table b).

In items 2 through 5 the reverse may also be found, i.e., given a probability, the level of exceedence corresponding to that probability can be found.

APPLICATIONS: Tidal probabilities can be used in cases where it is necessary to estimate the amount of time a given elevation will be exposed to wave action by virtue of the tide level. Examples include protective coatings on pier pilings, exposure of coastal structures to wave attack, and placement of beach fill. It is also possible to estimate the probability that vessels of a certain draft will be delayed while waiting for a more favorable tide before navigating a harbor entrance.

The design water level for a structure is usually based on Mean High Water. It is important to realize that 50% of the high water levels will exceed this level. If this poses a threat to the structure, a design level which will be exceeded less often may be found using the probability tables. In the same manner, 50% of the low water levels will be below Mean Low Water. Where scouring of a structure is likely to occur due to wave action at low tide levels, the probability tables may be used to obtain a design level that will be exposed less often.

TABLE - TIDE STATIONS WITH PROBABILITY STATISTICS

EASTPORT, MAINE	CHARLESTON, S.C.	SOUTH BEACH, OREG.
PORTLAND, MAINE	SAVANNAH RIVER ENTRANCE, GA.	ASTORIA, OREG.
BOSTON, MASS.	SAVANNAH, GA.	ABERDEEN, WASH.
NEWPORT, R.I.	MAYPORT, FLA.	PT. TOWNSEND, WASH.
NEW LONDON, CONN.	MIAMI HARBOR ENTRANCE, FLA	SEATTLE, WASH.
BRIDGEPORT, CONN.	KEY WEST, FLA.	FRIDAY HARBOR, WASH.
WILLETS POINT, N.Y.	NAPLES, FLA.	KETCHIKAN, ALASKA
NEW YORK (The Battery), N.Y.	ST. PETERSBURG, FLA.	JUNEAU, ALASKA
ALBANY, N.Y.	ST. MARKS RIVER ENTRANCE, FLA	SITKA, ALASKA
SANDY HOOK, N.J.	PENSACOLA, FLA.	CORDOVA, ALASKA
ATLANTIC CITY, N.J.	MOBILE, ALA.	SELDOVIA, ALASKA
BREAKWATER HARBOR, DEL.	GALVESTON (Ship Channel), TEX	ANCHORAGE, ALASKA
REEDY POINT, DEL	SAN JUAN, P.R.	KODIAK, ALASKA
PHILADELPHIA, PA.	SAN DIEGO, CALIF.	DUTCH HARBOR, ALASKA
BALTIMORE, MD.	LOS ANGELES (Outer Harbor), CALIF.	SWEEPER COVE, ALASKA
WASHINGTON, D.C.	SAN FRANCISCO (Golden Gate), CALIF.	MASSACRE BAY, ALASKA
HAMPTON ROADS, VA.	HUMBOLDT, CALIF.	NUSHAGAK, ALASKA
WILMINGTON, N.C.	CRESCENT CITY, CALIF.	ST. MICHAEL, ALASKA
		HONOLULU, HAWAII

The following examples illustrate the use of the tidal probability tables for a number of typical design applications. Other uses will be evident to the coastal engineer familiar with probability distributions.

********************************** EXAMPLE 1 ************************************

REQUIRED: The maximum and minimum predicted tidal levels expected to occur over a 19-year period at Newport, R.I.

SOLUTION: From the Index to Appendix B of SR-7 for Newport, R.I. (page 115) the Normalizing Factor (N.F.) is 1.810 feet (M). This value is half of the mean tide range. Note that in some cases N.F. is half of the diurnal range (D).

Maximum: From Table B-4a under the heading "Extreme Mo. High Water" (or Table B-4b under "Higher High Water", "High Water", or "Hourly Values") for Class No. 101, the Lower Limit (L.L.) is 1.8832.

The Maximum Tide Level = L.L. x N.F. = (1.8832)(1.810) = 3.41 feet (above Mean Sea Level)

Minimum: From Table B-4a under the heading "Extreme Mo. Low Water" for Class No. 1, the Lower Limit is -1.6223.

Therefore, the Minimum Tide Level = L.L. x N.F. = (-1.6223)(1.810) = -2.94 feet (relative to MSL).

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*************************************** EXAMPLE 2 ***********************************

PROBLEM: A weir sand-bypassing system at Mayport, FL. will not bypass sand when the tide level falls lower than 3.4 feet below MSL (-1.13 feet MLW).

REQUIRED: Estimate:

(a) The probability that a low tide will be below -3.4 feet during the month.

(b) The probability that a daily low water level will be below -3.4 feet.

(c) The hours per year that the system will not bypass sand.

SOLUTION:

(a) From the station index on page 115 of SR-7, the Normalizing Factor (N.F.) for Mayport is 2.232 feet (M). The required lower limit for use in the probability tables is found as

$$\text{Lower Limit} = \frac{\text{tide level}}{\text{N.F.}} = \frac{-3.4 \text{ feet}}{2.232 \text{ feet}} = -1.5233.$$

From Table B-22a under the column "Extreme Mo. Low Water" the Lower Limit falls between Class Nos. 34 and 33 as shown below.

The probability that a low tide level will be above -3.4 feet is found by linear interpolation using equation 28 from SR-7.

Class No.	Lower Limit	Cum. Freq.
34	-1.5214 (h_+)	0.6360 (P_+)
	-1.5233 (h_c)	$P(h>h_c)$
33	-1.5283 (h_-)	0.6491 (P_-)

$$P(h>h_c) = P_- + \left(\frac{h_c - h_-}{h_+ - h_-}\right)(P_+ - P_-)$$

Using the values tabulated to the right,

$$P(h>-3.4) = 0.6491 + \left(\frac{-1.5233 - (-1.5283)}{-1.5214 - (-1.5283)}\right)(0.6360 - 0.6491) = 0.6396.$$

The probability that a low water will be below -3.4 feet at some time during the month is (1 - 0.6396)(100%) = 36.0%.

(b) The probability that a daily low water level will be lower than -3.4 feet is found from Table B-22b of SR-7 under the column "Low Water". Using the same Lower Limit (L.L. = -1.5233) as part (a), and interpolating as before (Table at right) gives

$$P(h>-3.4) = 0.9703,$$

which is the probability that a daily low water level is above -3.4 feet MSL. The probability that a daily low water level will be below -3.4 feet is (1 - 0.9703)(100%) = 3.0%.

Class No.	Lower Limit	Cum. Freq.
16	-1.5178 (h_+)	0.9687 (P_+)
	-1.5233 (h_c)	$P(h>h_c)$
15	-1.5332 (h_-)	0.9731 (P_-)

(c) The average number of hours per year that the water level is lower than 3.4 feet below MSL is found from the column "Hourly Values" (Table B-22b of SR-7). Using the same Lower Limit and interpolating as before, $P(h>-3.4) = 0.9973$, and the hours per year that the system does not bypass sand is

$$(1 - 0.9973)(24 \text{ hrs/day})(365 \text{ days/yr}) = 23.7 \text{ hrs/yr}.$$

Class No.	Lower Limit	Cum. Freq.
8	-1.5038	0.9966
	-1.5233	
7	-1.5387	0.9978

NOTE: Time estimates should only be calculated using "Hourly Values".

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*** EXAMPLE 3 ***

REQUIRED: Estimate the tidal range at the Golden Gate, San Francisco, in which 90% of the high water levels and 90% of the low water levels occur each month.

SOLUTION *High Water Levels:*

When 90% of the monthly high waters are below a given level, then 10% of the high waters will exceed this level. The Cumulative Frequency of this exceedence is

$$\frac{10\%}{100\%} = 0.10 .$$

Class No.	Lower Limit	Cum. Freq.
87	1.3175 (h_+)	0.0965 (P_+)
	(h_c)	0.1000 ($P(h>h_c)$)
86	1.3129 (h_-)	0.1096 (P_-)

From Table B-34a of SR-7 under the column "Extreme Mo. High Water" for the Cumulative Frequency = 0.10, the value of h_c is interpolated between Class Nos. 87 and 86 using the formula

$$h_c = h_- + \left[\frac{(P(h > h_c) - P_-)}{(P_+ - P_-)}\right](h_+ - h_-) .$$

Using the values tabulated above,

$$h_c = 1.3129 + \left[\frac{(0.1000 - 0.1096)}{(0.0965 - 0.1096)}\right](1.3175 - 1.3129) = 1.3163 .$$

The elevation above MSL is found by multiplying the value of h_c by the Normalizing Factor given in Appendix B Index (N.F. = 2.865 feet).

$$+h_{90\%} = 1.3163\ (2.865) = 3.77 \text{ feet above MSL.}$$

Low Water Levels:

When 90% of the monthly low tides are above a given level, this level corresponds to a Cumulative Frequency of 0.90. From column "Extreme Mo. Low Water", Table B-34a of SR-7, interpolation gives

Class No.	Lower Limit	Cum. Freq.
14	-1.7061	0.8991
	(h_c)	0.9000
13	-1.7170	0.9079

$$h_c = -1.7072$$

and the elevation below MSL is

$$-h_{90\%} = -1.7072\ (2.865) = -4.89 \text{ feet (below MSL)}$$

The total tidal range which includes 90% of the monthly high waters and 90% of the monthly low waters is

Range = 3.77 ft. - (-4.89 ft.) = 8.66 feet.

The extreme predicted range over a 19-year period at the Golden Gate, San Francisco, (estimated as per Example 1) is

Extreme Tide Range = 3.962 - (-5.147) = 9.11 feet.

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NOTE: For approximate estimates interpolation is generally not necessary.

************************************** EXAMPLE 4 ********************************

PROBLEM: Vessels of a certain draft are sometimes delayed at the entrance to Humboldt Bay, CA. while awaiting sufficient tide levels in order to safely pass through the channel.

REQUIRED: A shoal in the channel has a depth of -30.5 feet MLLW (-34.0 feet MSL). A vessel drawing 26.0 feet requires a minimum depth of 5.5 feet under the keel (neglecting wave action) for a total required depth of 31.5 feet. This corresponds to a required tide level of -2.5 feet (relative to MSL) to provide the minimum required depth. Estimate the probability from "Hourly Values" that this vessel will be delayed at the entrance.

SOLUTION: The Normalizing Factor for Humboldt Bay (from Appendix B Index) is 3.202 feet (D). The required Lower Limit is

$$\text{L.L.} = \frac{-2.5 \text{ ft.}}{3.202 \text{ ft.}} = -0.7808.$$

Class No.	Lower Limit	Cum. Freq.
32	-0.7791	0.8741
	-0.7808	
31	-0.8123	0.8844

From Table B-35b under the column "Hourly Values," the Cumulative Frequency is interpolated between Class Nos. 32 and 31 give

$$P(h>-2.5) = 0.8746.$$

Therefore, the probability that the tide level will be below -2.5 feet (MSL) is (1 - 0.8746) = 0.1254 or 12.5% of the time.

NOTE: This type of information can be useful in determining design depth requirements for navigation projects.

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Note that when a ship must transit some distance into a harbor, and the ship is entering on an ebb tide, that the speed of tidal progression into the harbor may be greater than the speed of the ship. The progression of the tide is governed by low amplitude wave theory and can be estimated by $c = \sqrt{gd}$ where c is the celerity, g is gravitational acceleration (32.2 ft/sec^2), and d is the water depth in the estuary, bay, or sound (not in the dredged channel). If shallow areas exist in the channel inside the bar, then the percentage of time that vessels are delayed may be greater than the value calculated following the procedure in the above example.

************************* Example 5 ******************************

PROBLEM: It is sometimes necessary to estimate tide elevation probabilities at locations other than those listed in the Table. The following two examples illustrate how this can be done.

(a) <u>REQUIRED</u>. Adjust the tidal range estimate at Example 3 for a location near Oakland Airport (Oakland, CA).

<u>SOLUTION</u>: From Appendix C of SR-7 Oakland Airport is Station 533, which is referenced to the primary station -- San Francisco is 1/2 of the Diurnal Tide Range, the adjusted N.F. for Oakland Airport is half the Diurnal Range of Station 533. For the Diurnal Range of 6.5 feet, the N.F. becomes

$$N.F. = 6.5/2 = 3.25$$

Using the same values of Lower Limits found in Example 3, the tidal values for Oakland Airport become:

$$\text{High} = 1.3163(3.25) = 4.28 \text{ feet}$$
$$\text{Low} = -1.7072(3.25) = -5.55 \text{ feet}$$

and the tidal range which includes 90% of the High and Low Waters is

$$\text{Range} = 4.28 - (-5.55) = 9.83 \text{ feet}.$$

(b) <u>REQUIRED</u>: Determine for Fort George Inlet, FL, the probabilities requested in parts (a) and (c) of Example 2 using Mayport as the primary reference station.

<u>SOLUTION</u>: Since Mayport N.F. is based on Mean Tidal Range, the N.F. for Fort George is found from Station 2851 in Appendix C of SR-7 as

$$N.F. = 4.8/2 = 2.4$$

The required Lower Limit becomes:
Lower Limit = -3.4 ft/2.4 ft = -1.4167. The remainder of the Example is worked exactly like parts (a) and (c) of Example 2 using the Mayport Table B-22a.

(a) The result is $P(h>-3.4) = 0.3751$. Therefore, there is a 62.5 % probability that the low water level will be lower than 3.4 feet below MSL at Fort George Inlet during any month.

(c) Using the same value of Lower Limit = -1.4167, Table B-22b under the column "Hourly Values" gives the result: $P(h>-3.4) = 0.9917$, and the estimated hours per year that the system does not bypass sand is

$$(1 - 0.9917)(24 \text{ hrs/day})(365 \text{ days/year}) = 72.7 \text{ hrs/year}.$$

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<u>REFERENCE</u>:

Harris, D. L. 1981. "Tides and Tidal Datums in the United States," SR-7, US Army Corps of Engineers, Coastal Engineering Research Center, Vicksburg, MS.